if the as yet undetermined A be taken as $k!$.

Another interesting special case is found on treating

$$\frac{A}{(m-k)(m-k+1)\cdots(m-1)m(m+1)\cdots(m+k-1)(m+k)},$$

especially if k be allowed to become infinite (cotangent series).

Also solved by FRANK IRWIN, C. F. GUMMER, S. BEALTY, O. SCHMIEDEL, OLIVE C. HAZLETT, and RALPH KEFFER.

485 (Algebra). Proposed by J. L. WALSH, Madison, Wisconsin.

Is it true that to every convergent series of positive terms, $a_1 + a_2 + a_3 + \cdots$, there corresponds a series of the type

$$\frac{M}{1^p} + \frac{M}{2^p} + \frac{M}{3^p} + \cdots,$$

such that $M/k^p > a_k$, $p > 1$?

SOLUTION BY E. H. MOORE, The University of Chicago.

A convergent series of positive terms, $a_1 + a_2 + \cdots + a_k + \cdots$, is a positive-valued function α; $\alpha(k) = a_k$ $(k = 1, 2, 3, \cdots)$, of the variable positive integer k, satisfying the condition that the corresponding series is convergent. Denote the class of all such functions α by $\mathfrak{C}_+$.

If the question proposed is answered in the affirmative, then there exists a sequence α_n $(n = 1, 2, 3, \cdots)$ of functions α_n of the class $\mathfrak{C}_+$, viz., the sequence of functions

$$\alpha_n : \alpha_n(k) = \frac{n}{k^{1+(1/n)}} \quad (k = 1, 2, 3, \cdots),$$

of such a nature that every function α of the class $\mathfrak{C}_+$ is dominated by a suitably chosen function α_n of the sequence, viz., $|\alpha(k)| \leqq |\alpha_n(k)|$ $(k = 1, 2, 3, \cdots)$,—that is, the class $\mathfrak{C}_+$ has the dominance property D_2 defined (for the general class of real-valued functions on the general range) in § 22 of my *Introduction to a Form of General Analysis* (The New Haven Mathematical Colloquium, Yale University Press, 1910, p. 41).

Now I have proved (§ 23*c* (5), *loc. cit.*) that the class $\mathfrak{M}^{\mathrm{III}_1}$ (and, a fortiori, the class $\mathfrak{C}_+$) of all absolutely convergent series of real-valued terms fails to have the dominance property D_2. Hence, the question proposed must be answered in the negative.

The proposition cited is one of a number of theorems involving various dominance properties. The present question may be answered still more luminously by citing the theorem of Hadamard (Acta Mathematica, vol. 18, 1894, p. 328, theorem (β); cf. also *loc. cit.*, p. 49) that for every sequence (α_n) of functions of the class $\mathfrak{C}_+$ such that for every k $\alpha_n(k)$ increases with n there exists a function α of the class $\mathfrak{C}_+$ of such a nature that for every n

$$\lim_{k=\infty} \frac{\alpha_n(k)}{\alpha(k)} = 0.$$

Also solved by ELIJAH SWIFT.

486 (Algebra). Proposed by FLORENCE P. LEWIS, Baltimore, Md.

Find the condition which must be satisfied by the coefficients of the quartic

$$a_0x^4 + a_1x^3 + a_2x^2 + a_3x + a_4 = 0$$

in order that the equation be solvable by successive applications of the quadratic formula.

SOLUTION BY THE PROPOSER.

The equation must be of the form $A(ax^2 + bx + c)^2 + B(ax^2 + bx + c) + C = 0$ or $Ay^2 + By + C = 0$. Let x_1 and x_1' correspond to root y_1, and x_2 and x_2' to root y_2, respectively. We then have $x_1 + x_1' = -b/a$ and $x_2 + x_2' = -b/a$. This says that the line joining x_1 and x_1' is bisected at $-b/2a$, or that x_1 and x_1' are harmonic conjugates as to $-b/2a$ and ∞. The same

is true of x_2 and x_2'. Hence, ∞ is a root of the sextic covariant, whose leading coefficient must therefore vanish. This gives $8a_0^2a_3 - 4a_0a_1a_2 + a_1^3 = 0$ as the required condition. The same result may be readily obtained by elementary methods.

Also solved by HERBERT N. CARLETON, OTTO J. RAMLER, and HORACE OLSON.

E. B. ESCOTT sent in three solutions of 515 (Geometry). The first two solutions were solutions given in E. CATALAN'S *Théorèmes et Problèmes de Géométrie Élémentaire*, pp. 237–239. The third is a neat original solution.

516 (Geometry). Proposed by R. M. MATHEWS, Riverside, California.

Through the edges of a trihedral angle planes are passed orthogonal to the opposite faces Prove them coaxial.

I. SOLUTION BY NATHAN ALTSHILLER, University of Oklahoma.

The planes x, y, z which pass through the edges OA, OB, OC of the trihedral angle O–ABC and which are perpendicular to the opposite faces a, b, c, cut these faces along lines OD, OE, OF, respectively. A plane p perpendicular to OA cuts the edges OA, OB, OC in the points A, B, C, and the lines OD, OE, OF in the points D, E, F. (The reader may readily construct the figure.)

The plane x is perpendicular to a by construction, and to the plane p, because x passes through OA; hence, x is perpendicular to the line of intersection BC of p with a, and therefore BC is perpendicular to AD.

The plane b is perpendicular to y by construction and to the plane p, because b passes through OA; hence, b is perpendicular to the line of intersection BE of y with p, and, therefore, BE is perpendicular to AC. For similar reasons, CF is perpendicular to AB.

The three altitudes AD, BE, CF of the triangle ABC concur, according to a well-known proposition, in a point H, the orthocenter of ABC; hence H is a common point of the planes x, y, z. Now these three planes have obviously the point O in common, hence they pass through the line OH.

Incidentally we have also proved: *The locus of the orthocenter of the triangle determined by three concurrent lines and a variable plane perpendicular to one of the given lines is a straight line concurrent with the given lines.*

II. SOLUTION BY W. WOOLSEY JOHNSON, Annapolis, Md.

Referred to the sphere, the problem is that of the existence of the orthocenter of the spherical triangle ABC.

Let $CD = p$ be the perpendicular from C upon AB and let the perpendicular from A upon BC cut it in O.

From the right triangle AOD we have

$$\tan OD = \tan OAD \sin AD. \tag{1}$$

From AEB, we have

$$\cot OAD = \cos c \tan B. \tag{2}$$

From ADC, we have

$$\sin AD = \tan p \cot A. \tag{3}$$

Dividing (3) by (2) and substituting in (1), we have $\tan OD = \tan p \cot A \cot B \sec c$.

Interchanging A and B, c and p remain unchanged; hence, the perpendicular from B cuts off from p the same segment OD or the three perpendiculars meet in a common point.

Also solved by HORACE OLSON, L. E. LUNN, C. J. PAYNE, WILLIAM HOOVER, and the PROPOSER.